Adaptive Affinity Matrix for Unsupervised Metric Learning

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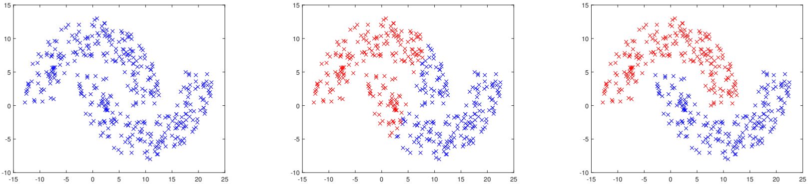
Background: Spectral Clustering

- Spectral clustering: nonlinear feature reduction.
- The distribution of real data does not always obey uniform or gaussian.
- Spectral clustering can preserve the local neighborhood information.



(a) (b) (c)

Background: Spectral Clustering

- Spectral clustering demonstrates a splendid performance on many challenge data sets.
- Objective function:

$$y = \arg\min_{y^T D y = 1} \sum_{i,j}^{n} w_{ij} \|y_i - y_j\|_2^2 ,$$

  where w_{ij} is the similarity between data sample x_i and x_j (a.k.a. affinity graph).
- Shortcomings of spectral clustering
  - Out-of-sample extension is not straightforward
  - Cubic time complexity
  - Sensitive to the affinity graph

Background: Locality Preserving Projections

- Locality Preserving Projections (LPP) [HN04] is the linear approximation of Laplacian Eigenmap.
- Locality Preserving Projections conducts dimensionality reduction by solving the optimization problem:

$$a = \underset{a^T XDXa=1}{\arg\min} \sum_{i,j}^{n} w_{ij} \| a^T x_i - a^T x_j \|_2^2 \ ,$$

- The superiority of LPP
  - Explicit projection for out-of-sample extension
  - Complexity is reduced

Motivation

- The performance of spectral clustering methods highly depends on the robustness of the affinity graph.
- Some weighting methods like $k-$NN heat kernel will be corrupted by noises.
- Our goal:
  - Learn a robust affinity graph by optimization efficiently.
  - Optimize the linear projection and affinity graph simultaneously.

Related Works

- Dominant Neighbors [PP07] reduces the noise of the affinity matrix by maximal cliques.
- Consensus k-NNs [PK13] builds affinity graph by consensus information.
- ClustRF-Strct [ZLG14] constructs an affinity graph via the clustering random forests.
- CAN and PCAN [NWH14] learn data similarity and cluster structure simultaneously.

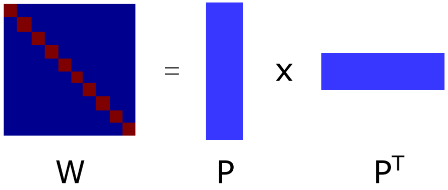
AdaAM: Assumption

- Assumption 1: The affinity matrix W is a positive semidefinite matrix. Hence we have,

$$W = PP^T .$$

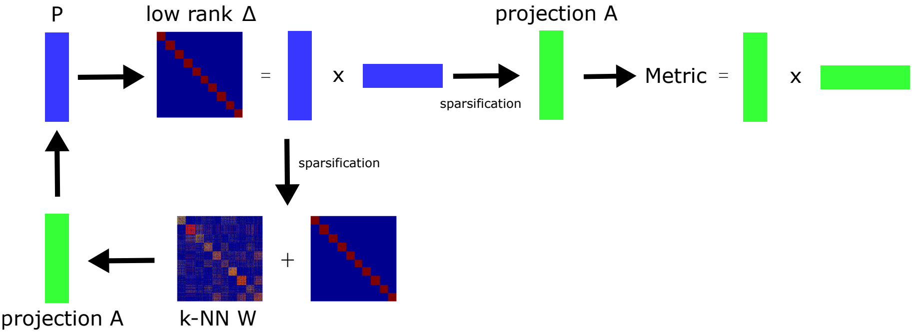
  This assumption also appeared in [CC11]

- Assumption 2: The ideal affinity matrix W is a low rank matrix (1 for the sample in the same class and 0 for the others).



AdaAM: Diagram

- A glance of our algorithm



AdaAM: Intermediate Affinity Matrix Δ

- Let Δ be the intermediate affinity matrix, and assume $\Delta = PP^T$. Compute P by solving optimization problem

$$\min_{P^TP=I} tr(X^T(D_\Delta - PP^T)X)$$
$$\Rightarrow \min_{P^TP=I} tr(X^T D_\Delta X) + tr(X^T(-PP^T)X)$$

  similar to spectral clustering

- When X is normalized with zero mean, we have $D_\Delta = \mathbf{0}$. The above problem is equivalent to

$$P = \arg\max_{P^TP=I} \; tr(P^T X X^T P)$$

AdaAM: Final Adaptive Affinity Matrix

- With the intermediate affinity matrix Δ, we can solve the following problem for a linear projection A:

$$A = \arg\min_{A^T A = I} tr(A^T X^T (L + L_\Delta) X A)$$

  $L + L_\Delta$ is the combination of the Laplacian of k-NN heat kernel and the intermediate affinity matrix.

- With the linear projection A, we can rewrite the affinity optimization problem and update matrix P ($D_\Delta = \mathbf{0}$ still holds).

$$P = \arg\max_{P^T P = I} tr(P^T X A A^T X^T P)$$

Experiments

- We evaluate the proposed approach on five image data sets
  - UMIST, COIL20, USPS, MNIST, ExYaleB
- We impose the same parameter selection criteria on all the algorithms in our experiments.
  - the size of neighborhood $k = \mathrm{Round}(\log_2(n/c))$
  - projected dimension is the same as the number of classes
- We denote 10 times of k-Means as a round and select the clustering result with the minimal within-cluster sum as the result of each round of k-Means.

Accuracy

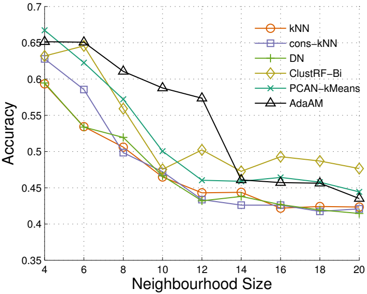
- 100 rounds *k*-Means to each algorithms for the evaluation of the performance.

Table: Clustering accuracy on image data sets(%)

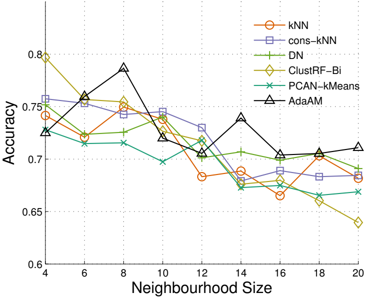
	AdaAM		***k*-NN**		**Cons-*k*NN**		**DN**		**ClustRF-Bi**		**PCAN-*k*Means**		**PCAN**
	Avg	Max	Avg	Max	Avg	Max	Avg	Max	Avg	Max	Avg	Max	
UMIST	**66.06**	**75.65**	58.16	65.39	60.27	69.22	59.15	66.96	64.63	74.44	53.79	56.52	55.30
COIL20	74.72	**87.29**	71.89	81.18	75.53	84.31	71.95	82.01	**76.50**	85.07	72.28	83.75	81.74
USPS	**69.36**	**69.61**	68.25	68.35	68.21	68.34	68.08	68.31	58.74	65.90	64.04	67.95	64.20
MNIST	**60.84**	**61.34**	48.13	48.27	47.88	48.00	49.72	49.76	51.93	52.03	58.93	58.98	59.83
ExYaleB	**54.36**	**57.87**	24.17	26.76	25.63	28.75	24.21	27.42	23.10	26.43	25.74	27.63	25.89

Accuracy

- 10 rounds *k*-Means for the experiment of the sensitivity to the neighborhood size



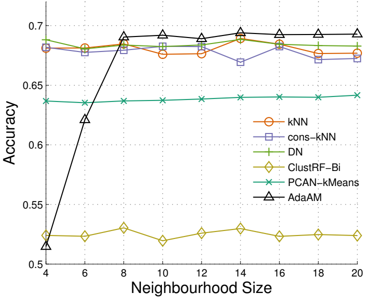
(d) UMIST



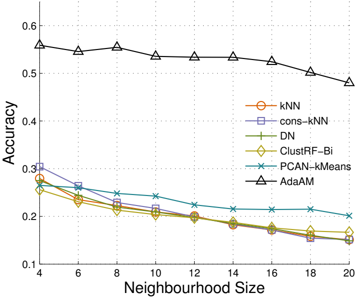
(e) COIL20

Figure: Comparison between different with different of neighborhood size *k*

Accuracy



(a) USPS



(b) ExYaleB

Figure: Comparison between different with different of neighborhood size k

- Requires more information from the pairwise similarity.
- For small k, sometimes does not perform well.

Time Consumption

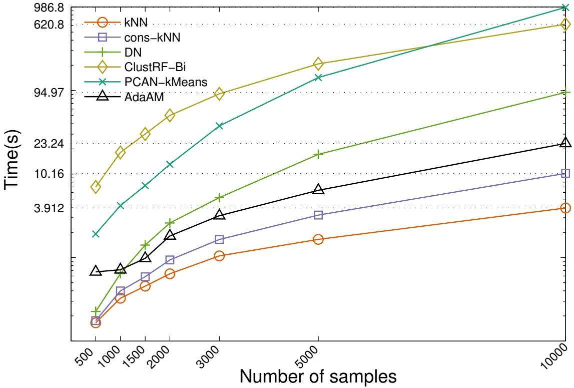


Figure: Time consumption of six approaches with different number of data instances

Conclusion & Future Work

- Conclusion
  - We present a novel affinity learning approach for unsupervised metric learning.
  - The affinity matrix is learned from the same framework of spectral clustering.
  - The affinity learning can be reduced to a singular value decomposition problem.
  - We employ the low rank trick to make our approach more efficient.
- Future Work
  - A better way to learn the parameter of sparsification
  - A better way to fuse low rank Δ and k-NN W.
  - More applications

Thanks

Thanks for your Attention.

References

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