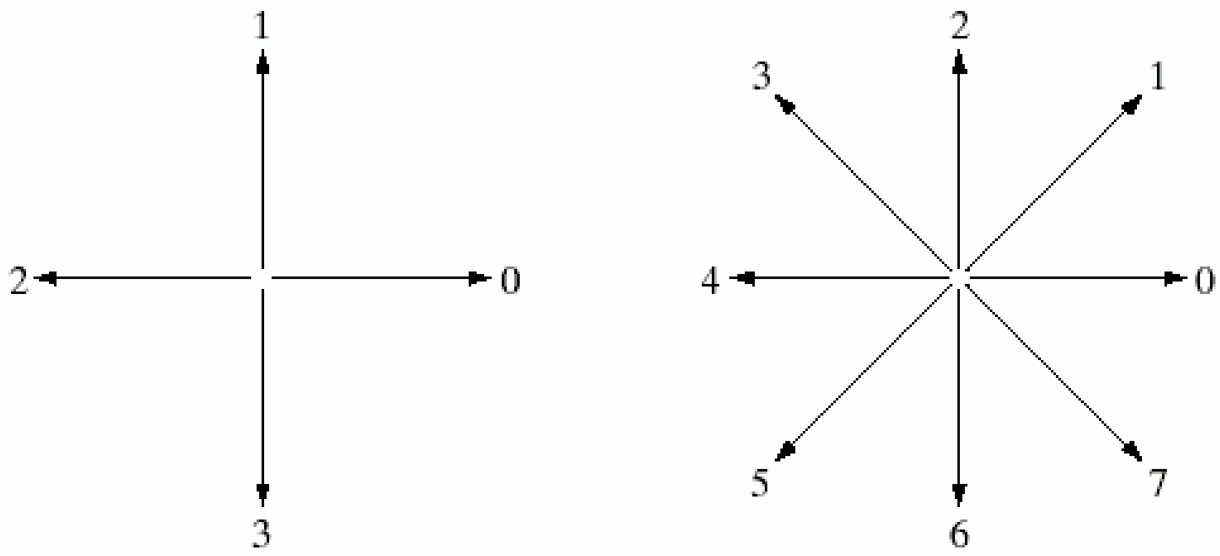
Chapter 11
Representation & Description



a b

FIGURE 11.1 Direction numbers for (a) 4-directional chain code, and (b) 8-directional chain code.

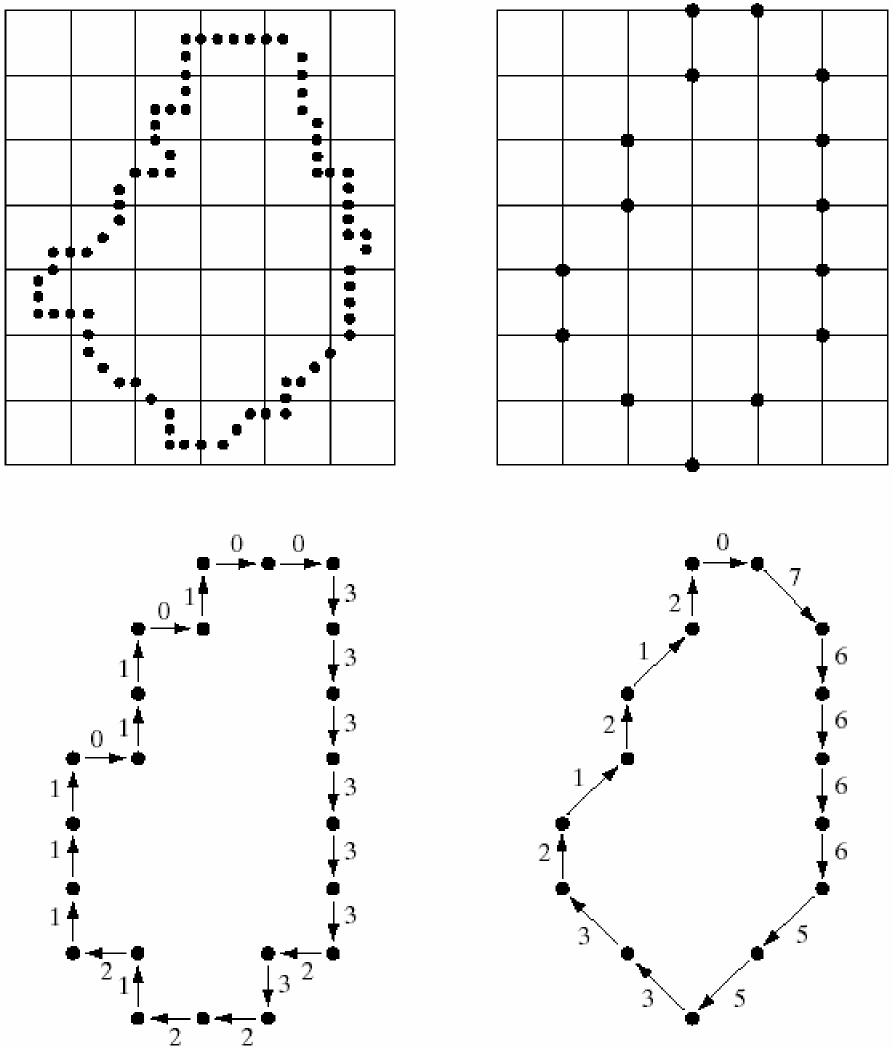
Chain Codes

- Chain codes are used to represent a boundary by a connected sequence of straight-line segments of specified length and direction.
- The direction of each segment is coded by using a numbering scheme such as the ones shown in Fig11.1.

Chain Codes

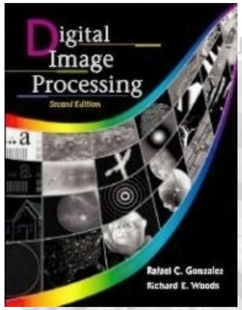
- The method generally is unacceptable for two principal reasons:
  - (1) The resulting chain of codes tends to be quite ling and,
  - (2) any small disturbances along the boundary due to noise or imperfect segmentation cause changes in the code that may not be related to the shape of the boundary.
- An approach frequently used to circumvent the problem just discussed is to resample the boundary by selecting a larger grid spacing, as illustrated in Fig 11.2(a).

Chapter 11
Representation & Description



a b
c d

FIGURE 11.2
(a) Digital boundary with resampling grid superimposed.
(b) Result of resampling.
(c) 4-directional chain code.
(d) 8-directional chain code.



Chain Codes

- The chain code of a boundary depends on the starting point.
- We can normalize also for rotation by using the *first difference* of the chain code instead of the code itself.
- The first-difference of the 4-direction chain code 10103322 is 3133030.
- If we elect to treat the code as a circular.
- Here, the result is 33133030.

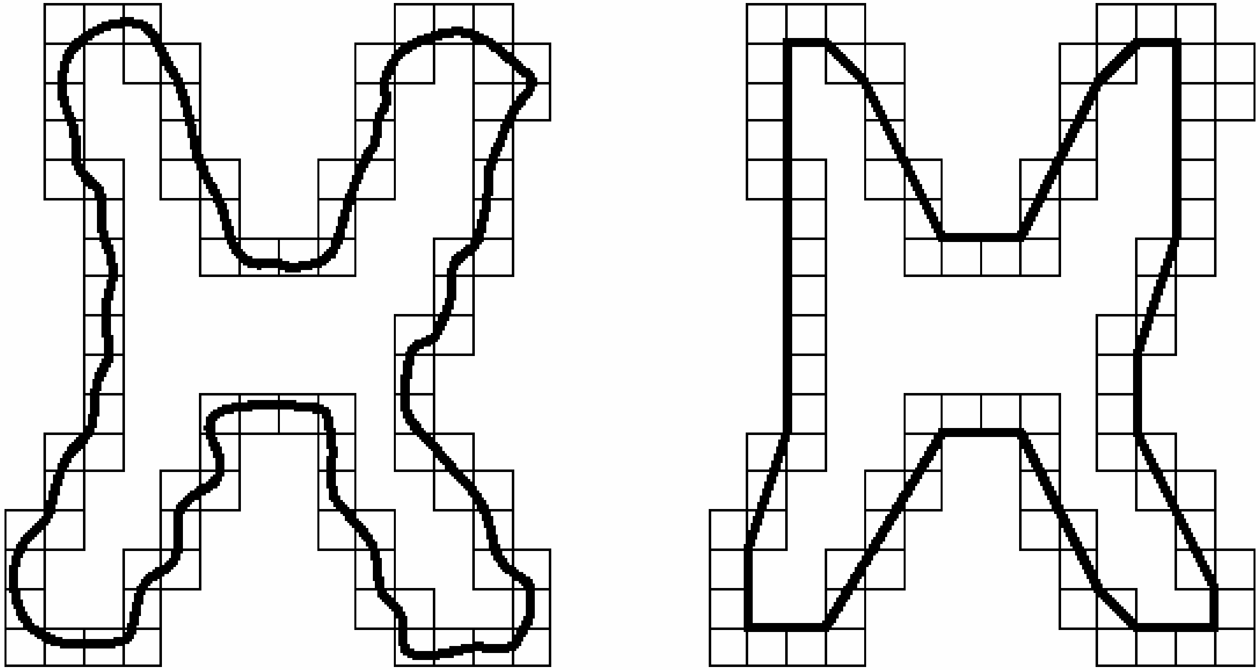
Polygonal Approximations

- A digital boundary can be approximated with arbitrary accuracy by a polygon.
- In practice, the goal of polygonal approximation is to capture the “essence” of the boundary shape with the fewest possible polygonal segments.

Minimum perimeter polygons

- Fig. 11.3(b), producing a polygon of minimum perimeter that fits the geometry established by the cell strip.

Chapter 11
Representation & Description



a b

FIGURE 11.3 (a) Object boundary enclosed by cells. (b) Minimum perimeter polygon.

Splitting techniques

- One approach to boundary segment splitting is to subdivide a segment successively into two part until a specified criterion is satisfied.
- For a closed boundary, the best starting points usually are two farthest points in the boundary.
- Fig 11.4(c) shows the result of using the splitting procedure with a threshold equal to 0.25 times the length of line *ab*.

Chapter 11
Representation & Description

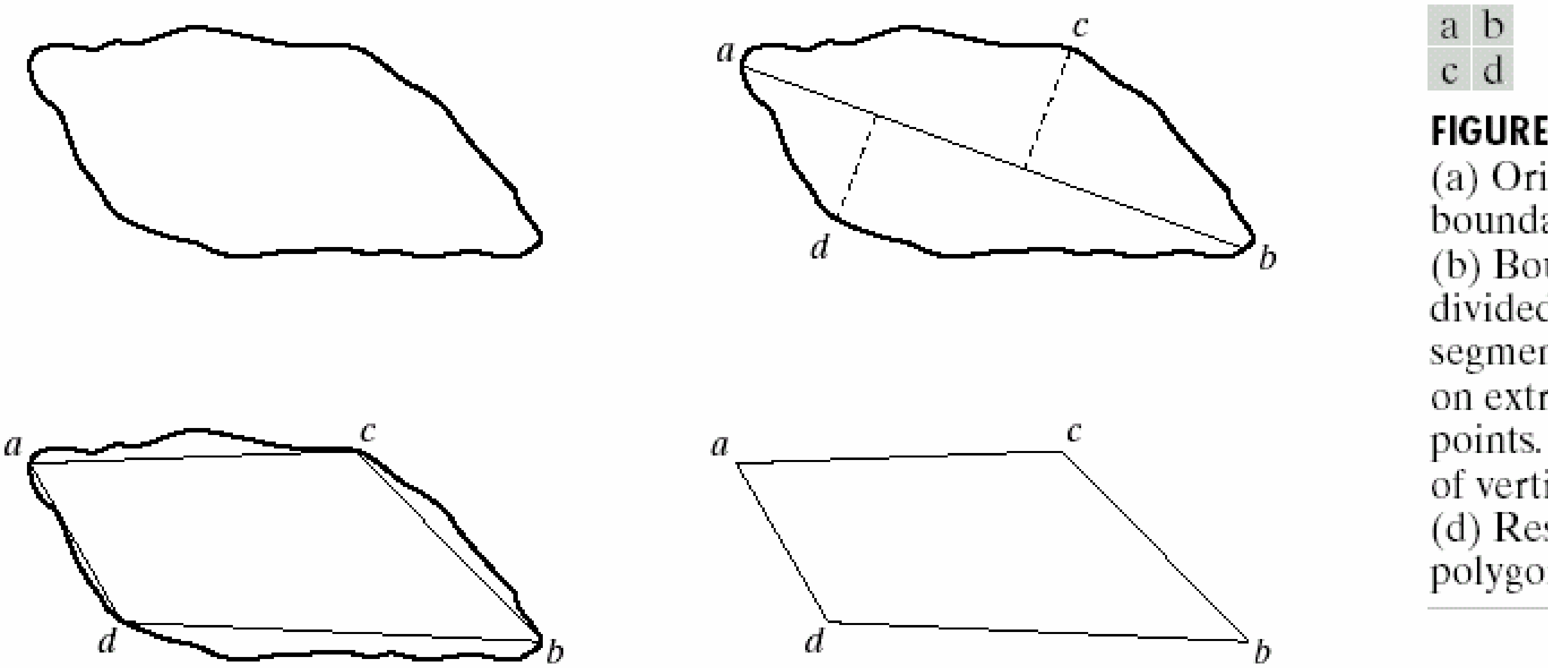
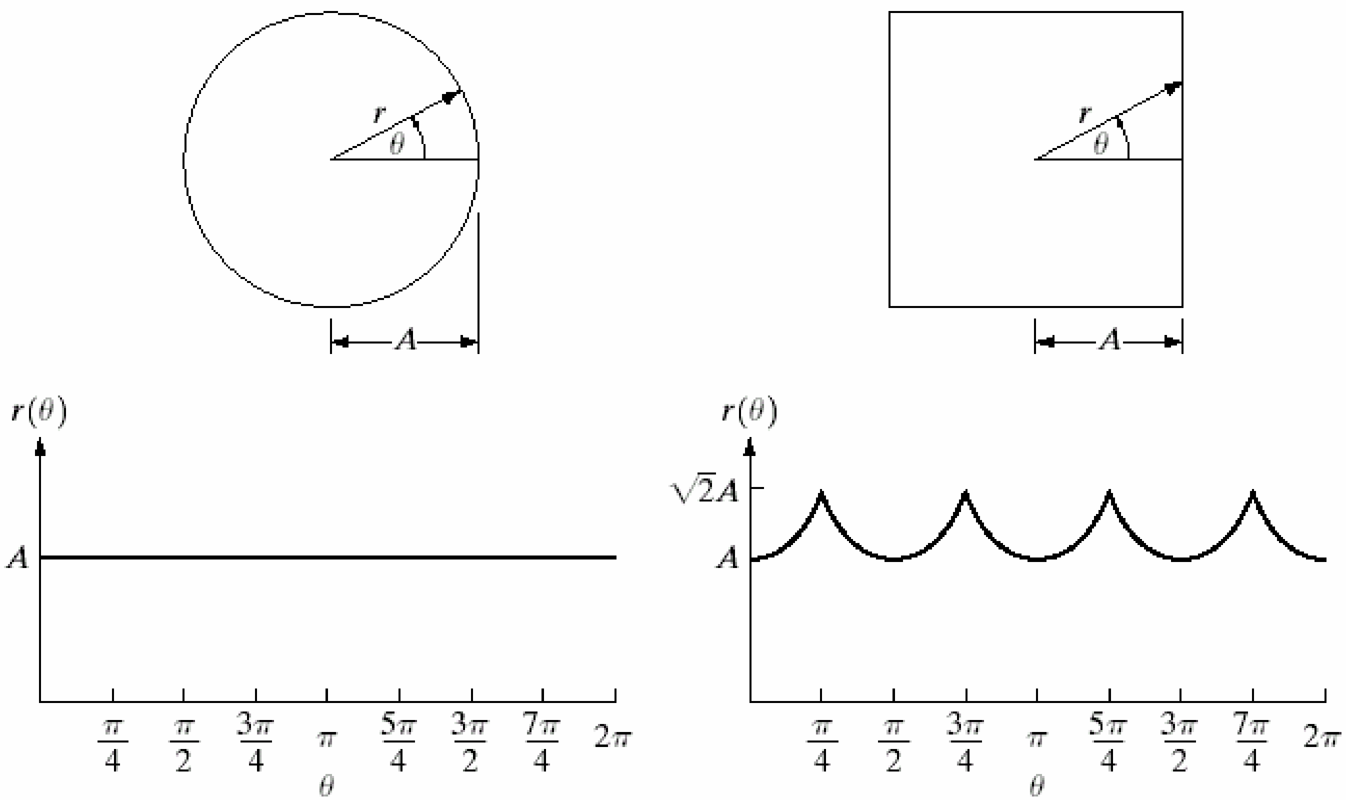


FIGURE 11.4 (a) Original boundary. (b) Boundary divided into segments based on extreme points. (c) Joining of vertices. (d) Resulting polygon.

Signatures

- A signature is a 1-D functional representation of a boundary and may be generated in various ways.
- One of the simplest is to plot the distance from the centroid to the boundary as a functoon of angle, as illustrated in Fig 11.5.

Chapter 11
Representation & Description



a b

FIGURE 11.5 Distance-versus-angle signatures. In (a) $r(\theta)$ is constant. In (b), the signature consists of repetitions of the pattern $r(\theta) = A \sec\theta$ for $0 \leq \theta \leq \pi/4$ and $r(\theta) = A \csc\theta$ for $\pi/4 < \theta \leq \pi/2$.

Signatures

- Signatures generated by the approach just described are invariant to translation, but they do depend on rotation and scaling.
- Normalization with respect to rotation can be achieved by finding a way to select the same starting centroid.
- Another way us to select the point in the eigen axis.

Signatures

- One way to normalize for this result is to scale all functions so that they always span the same range of values, say, [0,1].
- Whatever the method used, keep in mind that the basic idea is to remove dependency on size while preserving the fundamental shape of the waveforms.

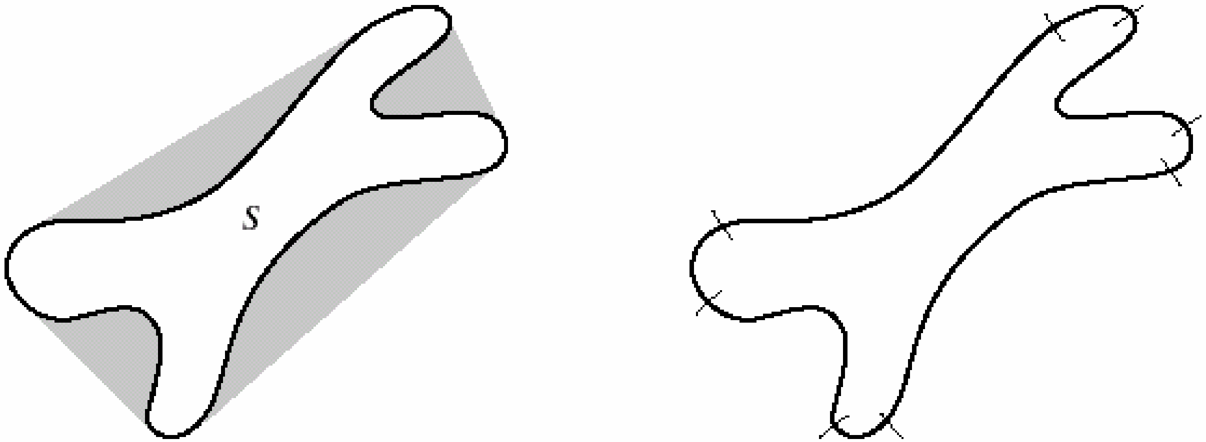
Boundary Segments

- Decomposing a boundary into segments often is useful.
- Decomposition reduces the boundary's complexity and thus simplifies the description process.
- In this case use of the convex hull of the region enclosed by the boundary is a powerful tool for robust decomposition of the boundary.

Boundary Segments

- *Convex hull* H of an arbitrary set *S* is the smallest convex set containing *S*.
- The set difference *H-S* is called the *convex deficiency D* of the set *S*.
- Fig 11.6(b) shows the result in this case.
- Note that in principle, this scheme is independent of region size and orentation.

Chapter 11
Representation & Description



a b

FIGURE 11.6 (a) A region, *S*, and its convex deficiency (shaded). (b) Partitioned boundary.

Skeletons

- An important approach to representing the structural shape of a plane region is to reduce it to a graph.
- This reduction may be accomplished by obtaining the *skeleton* of the region via a thinning (also called *skeletonizing*).

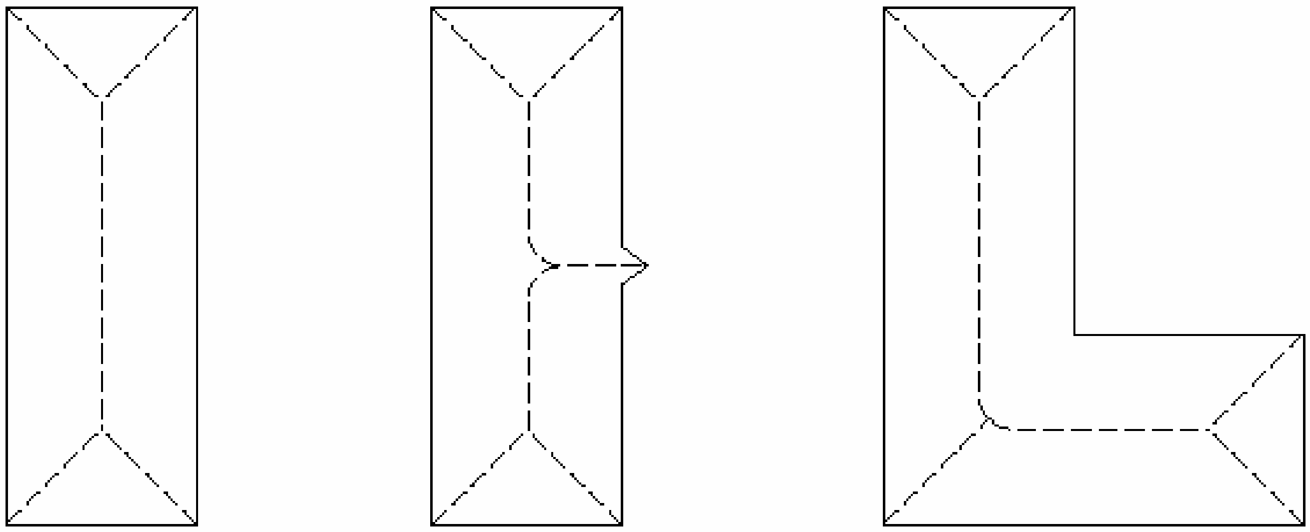
Skeletons

- The skeleton of a region may be defined via the medial axis transformation (MAT) proposed by Blum[1967].
- The MAT of a region has an intuitive definition based on the so-called “prairie fire concept.”
- Consider an image region as a prairie of uniform, dry grass, and suppose that a fire is lit along its border.
- All fire fronts will advance into the region at the same speed.
- The MAT of the region is the set of points reached by more than one fire front at the same time.

Skeletons

- Numerous algorithms have been proposed for improving computational efficiency.
- Typically, these are thinning algorithm that iteratively delete edge points of a region subject to the constraints that deletion of these points:
  - (1)does not remove end points
  - (2)does not break connectivity
  - (3)does not cause excessive erosion of the region.

Chapter 11
Representation & Description



a b c

FIGURE 11.7 Medial axes (dashed) of three simple regions.

Skeletons

- With reference to the 8-neighborhood notation shown in Fig 11.8, step 1 flags a contour point p_1 for deletion if the following conditions are satisfied:

(a) $2 \le N(p_1) \le 6$

(b) $T(p_1) = 1$

(c) $p_2 \cdot p_4 \cdot p_6 = 0$

(d) $p_4 \cdot p_6 \cdot p_8 = 0$

where $N(p_1)$ is the number of nonzero neighbors of p_1; that is

$N(p_1) = p_2 + p_3 + \cdots + p_8 + p_9$

Chapter 11
Representation & Description

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

FIGURE 11.8 Neighborhood arrangement used by the thinning algorithm.

Skeletons

- T(p_1) is the number of 0-1 transitions in the ordered sequence $p_2, p_3, \ldots, p_8, p_9, p_2$.
- For example, N(p_1)=4 and T(p_1)=3 in Fig 11.9.
- In step 2, conditions (a) and (b) remain the same, but conditions (c) and (d) are changed to

$$\text{(c')} \quad p_2 \cdot p_4 \cdot p_8 = 0$$

$$\text{(d')} \quad p_2 \cdot p_6 \cdot p_8 = 0$$

Skeletons

- Step 1 is applied to every border pixel in the binary region under consideration.
- If all conditions are satisfied the point is flagged for deletion.
- However, the point is not deleted until all border points have been processed.
- After step 1 has been applied to all border points, those that were flagged are deleted (changed to 0).
- Then step 2 is applied to the resulting data in exactly the same manner as step 1.
- The basic procedure is applied iteratively until no further points are deleted.

Chapter 11
Representation & Description

0	0	1
1	p_1	0
1	0	1

FIGURE 11.9 Illustration of conditions (a) and (b) in Eq. (11.1-1). In this case $N(p_1) = 4$ and $T(p_1) = 3$.

Chapter 11
Representation & Description

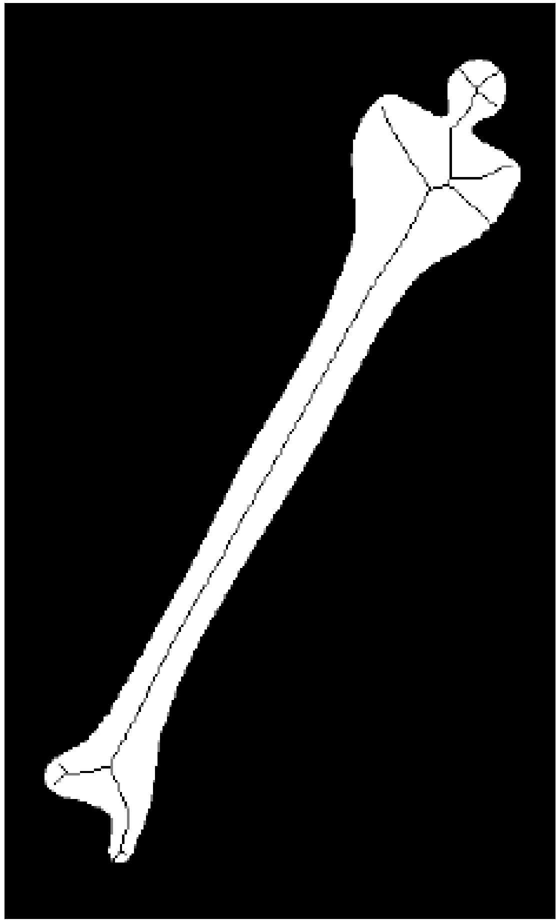


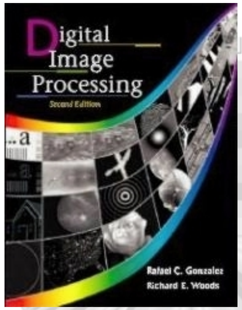
FIGURE 11.10 Human leg bone and skeleton of the region shown superimposed.

Some Simple Descriptors

- The *length* of a boundary is one of its simplest descriptors.
- The *diameter* of a boundary B us defined as

$\mathrm{Diam(B)} = \max_{i,j}\left[D\left(p_i, p_j\right)\right]$,this line is so-called the *major axis* of the boundary.

- The minor axis of a boundary is defined as the line perpendicular to the major axis.



Shape Numbers

- As explained in Section 11.1.1, the first difference of a chain-coded boundary depends on the starting point.
- The *shape number* of such a boundary, bssed on the 4-directional code of Fig 11.1(a), is defined as the first difference of smallest magnitude.
- The *order n* of a shape number is defined as the number of digits in its representation..
- Moreover, *n* is even for a closed boundary.

Chapter 11
Representation & Description

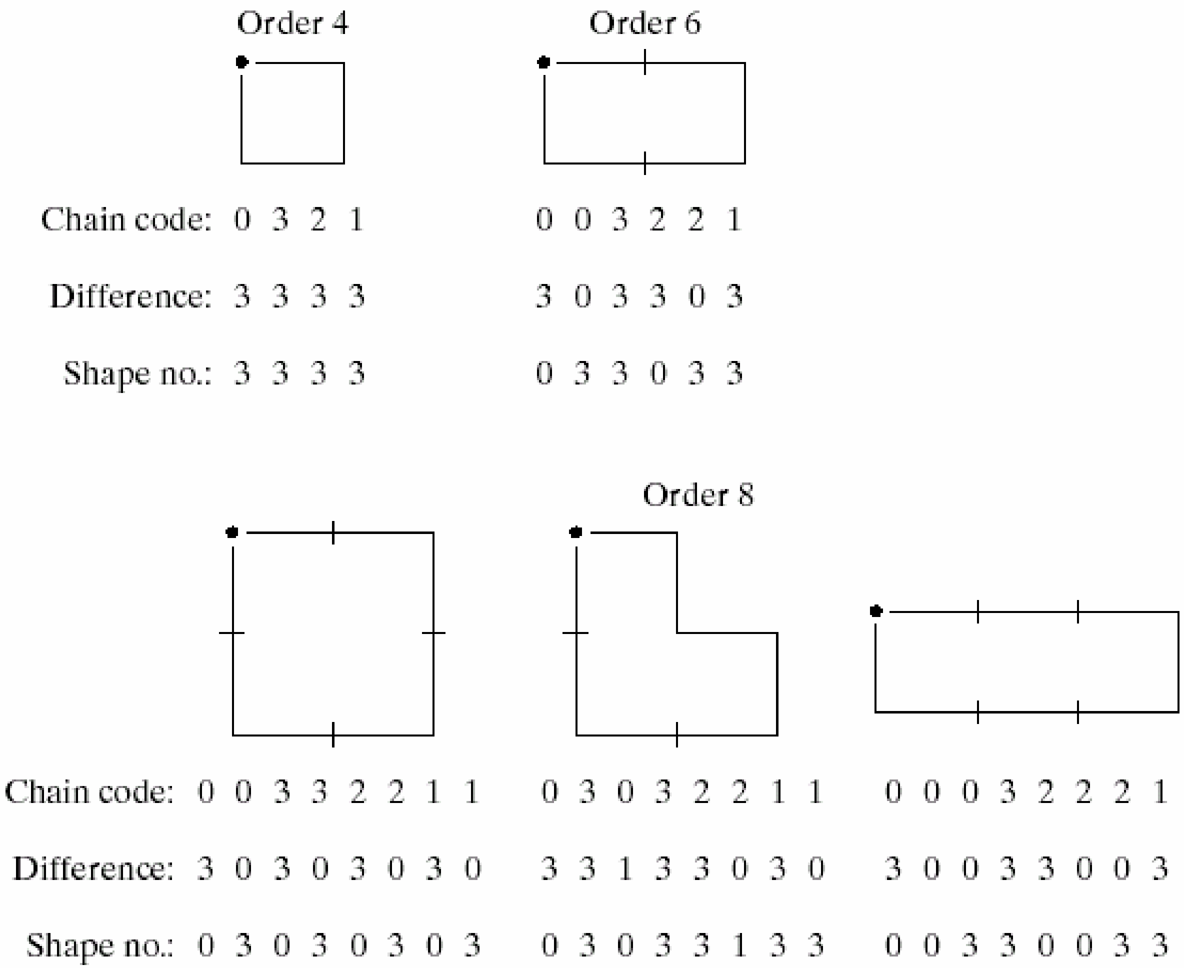
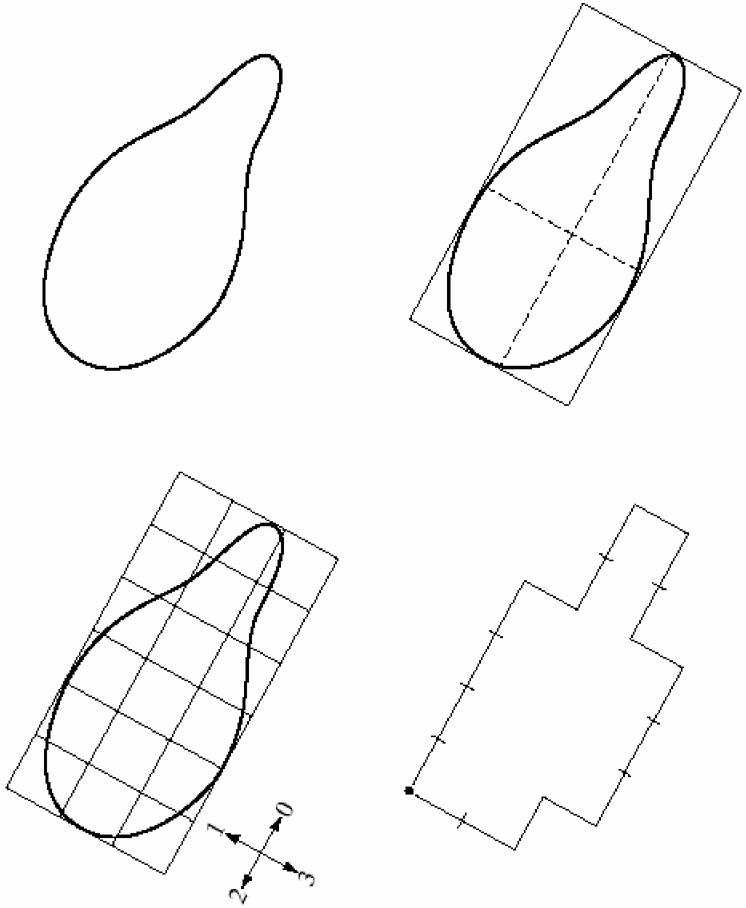


FIGURE 11.11 All shapes of order 4, 6, and 8. The directions are from Fig. 11.1(a), and the dot indicates the starting point.

Chapter 11
Representation & Description



a b
c d

FIGURE 11.12 Steps in the generation of a shape number.

Chain code:	0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1
Difference:	3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0
Shape no.:	0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3

Fourier Descriptors

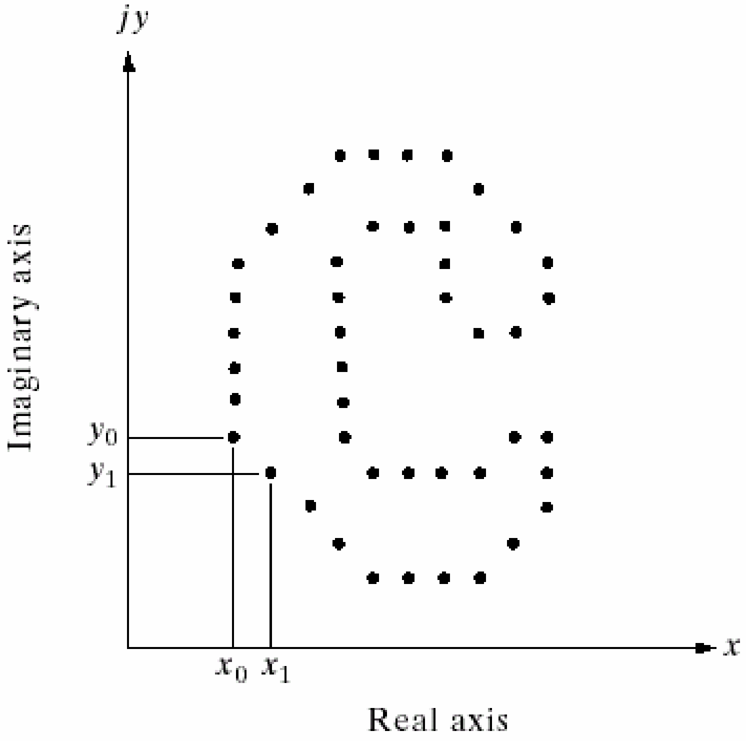
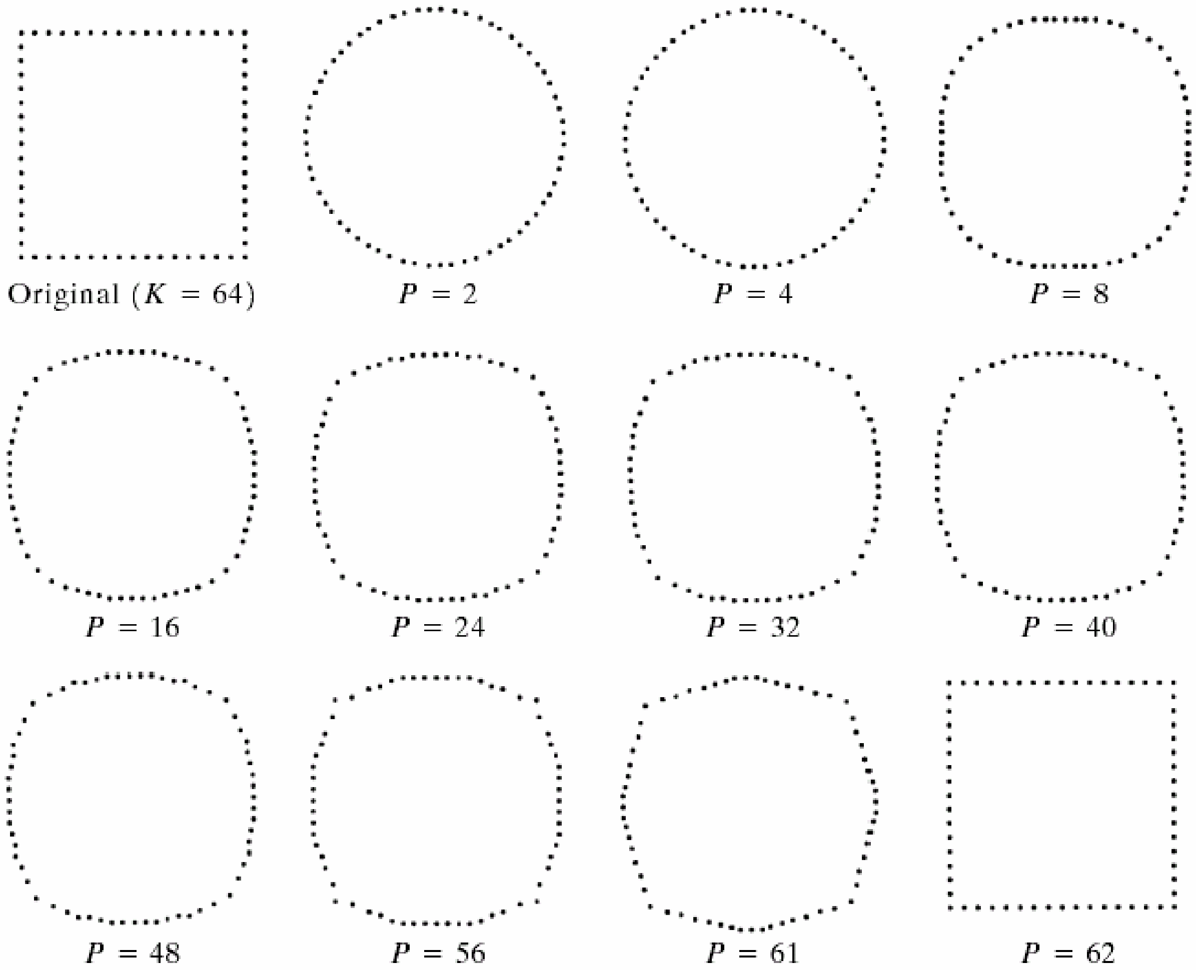


FIGURE 11.13 A digital boundary and its representation as a complex sequence. The points (x_0, y_0) and (x_1, y_1) shown are (arbitrarily) the first two points in the sequence.

Chapter 11
Representation & Description

FIGURE 11.14 Examples of reconstruction from Fourier descriptors. P is the number of Fourier coefficients used in the reconstruction of the boundary.



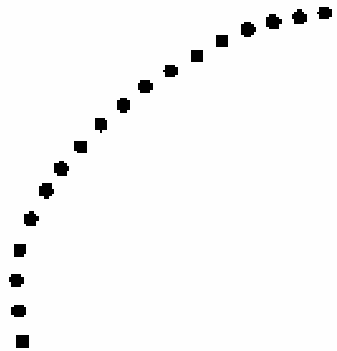
Chapter 11
Representation & Description

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

TABLE 11.1 Some basic properties of Fourier descriptors.

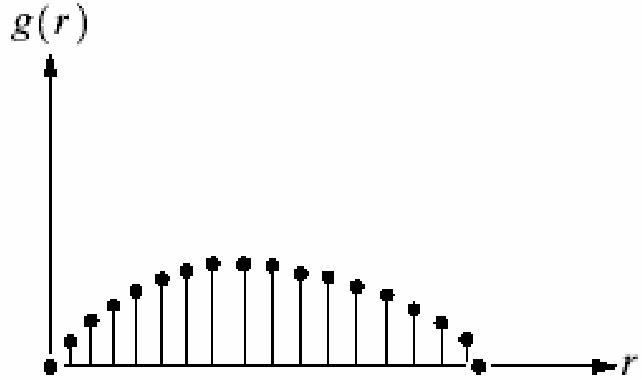
Statistical Moments

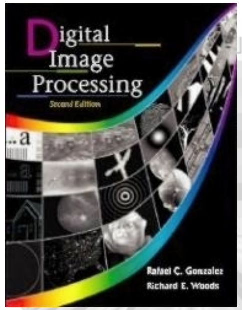
Chapter 11
Representation & Description



a b

FIGURE 11.15
(a) Boundary segment.
(b) Representation as a 1-D function.





Some simple Descriptors

- The area of a region is defined as the number of pixels in the region.
- The perimeter of a region is the length of its boundary.
- Compactness of a region, defined as (perimeter)^2/area, and is minimal for a disk-shape region.
- Other simple measures used as region descriptors include the mean and median of the gray levels, the minimum and maximum gray-level values, and the number of pixels with above and below the mean.

Topological Descriptors

- If a topological descriptors is defined by the number of holes in the region, this property obviously will not be affected by a stretching or rotation transformation.
- Another topological property useful for region description is the number of connected components.
- The number of holes *H* and connected components *C* in a figure can be used to define the *Euler Number E*: $E=C-H$

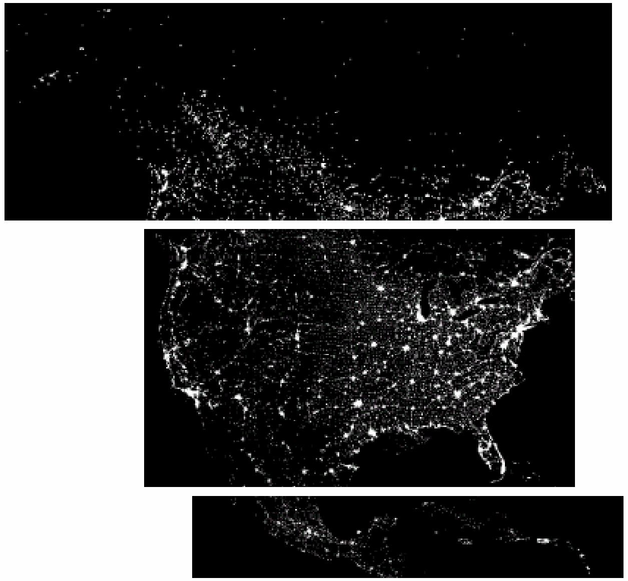
Topological Descriptors

- The Euler number is also a topological property.
- The regions shown in Fig 11.19, for example, have Euler numbers equal to 0 and -1.
- Denoting the number of vertices by *V*, the number of edges by *Q*, and the number of faces by *F* gives the following relationship, called the *Euler Formula*:

$$V - Q + F = C - H$$

$$= E.$$

Chapter 11
Representation & Description



Region no, (from top)	Ratio of lights per region to total lights
1	0.204
2	0.640
3	0.049
4	0.107

FIGURE 11.16 Infrared images of the Americas at night. (Courtesy of NOAA.)

Chapter 11
Representation & Description

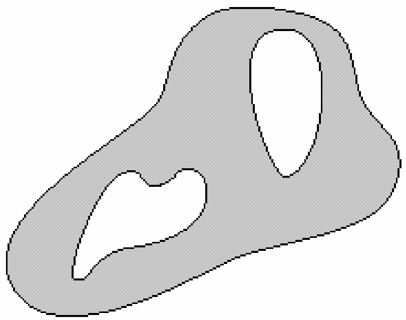


FIGURE 11.17 A region with two holes.

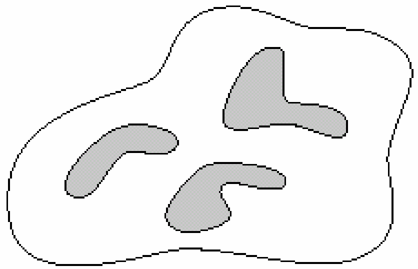
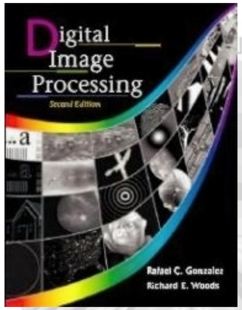
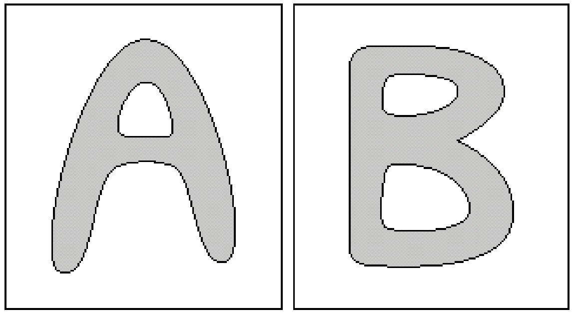


FIGURE 11.18 A region with three connected components.



Chapter 11
Representation & Description



a b

FIGURE 11.19 Regions with Euler number equal to 0 and −1, respectively.

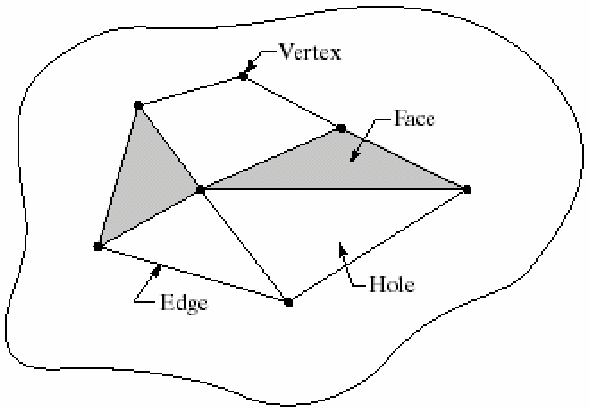
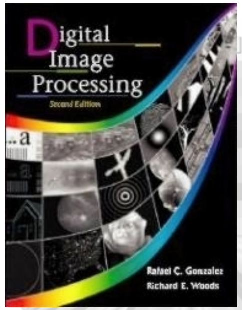
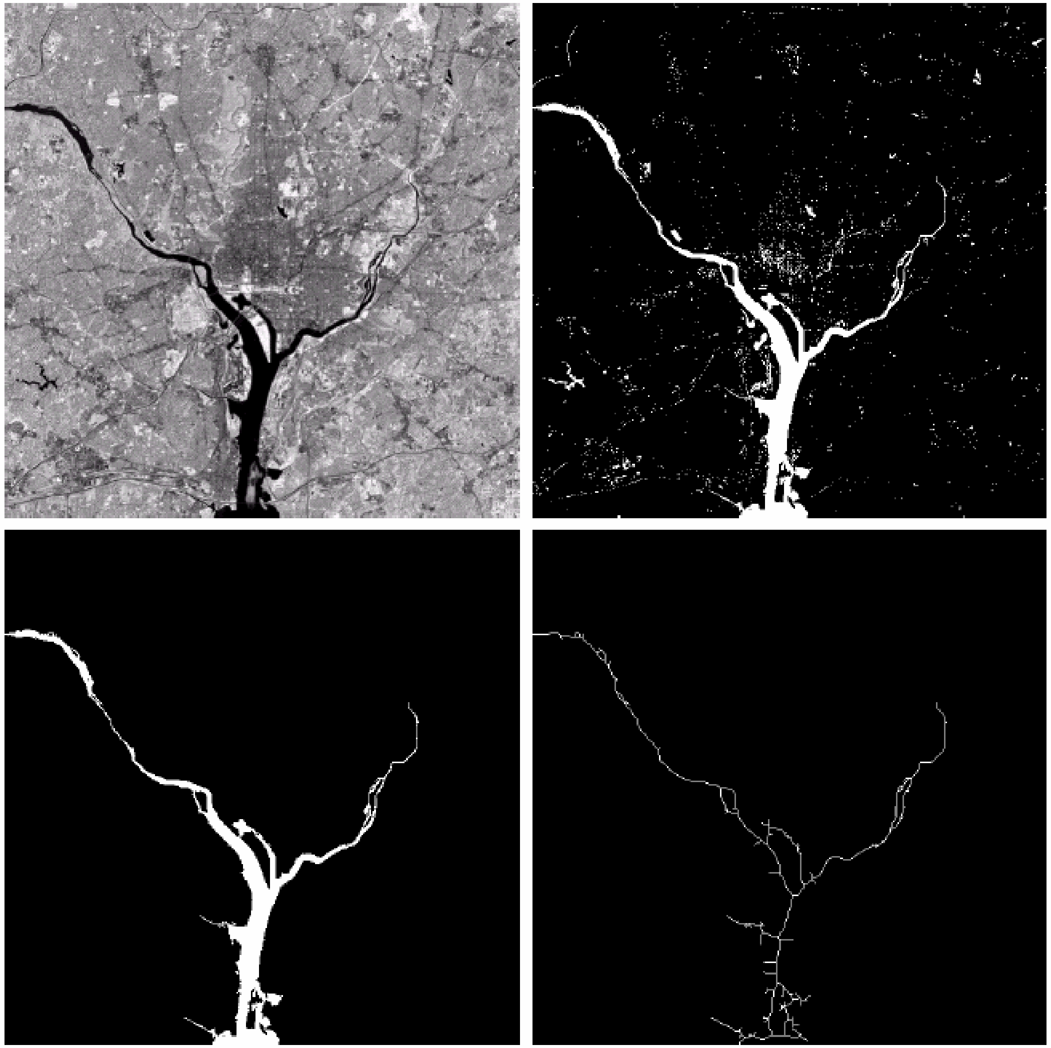


FIGURE 11.20 A region containing a polygonal network.



Chapter 11
Representation & Description



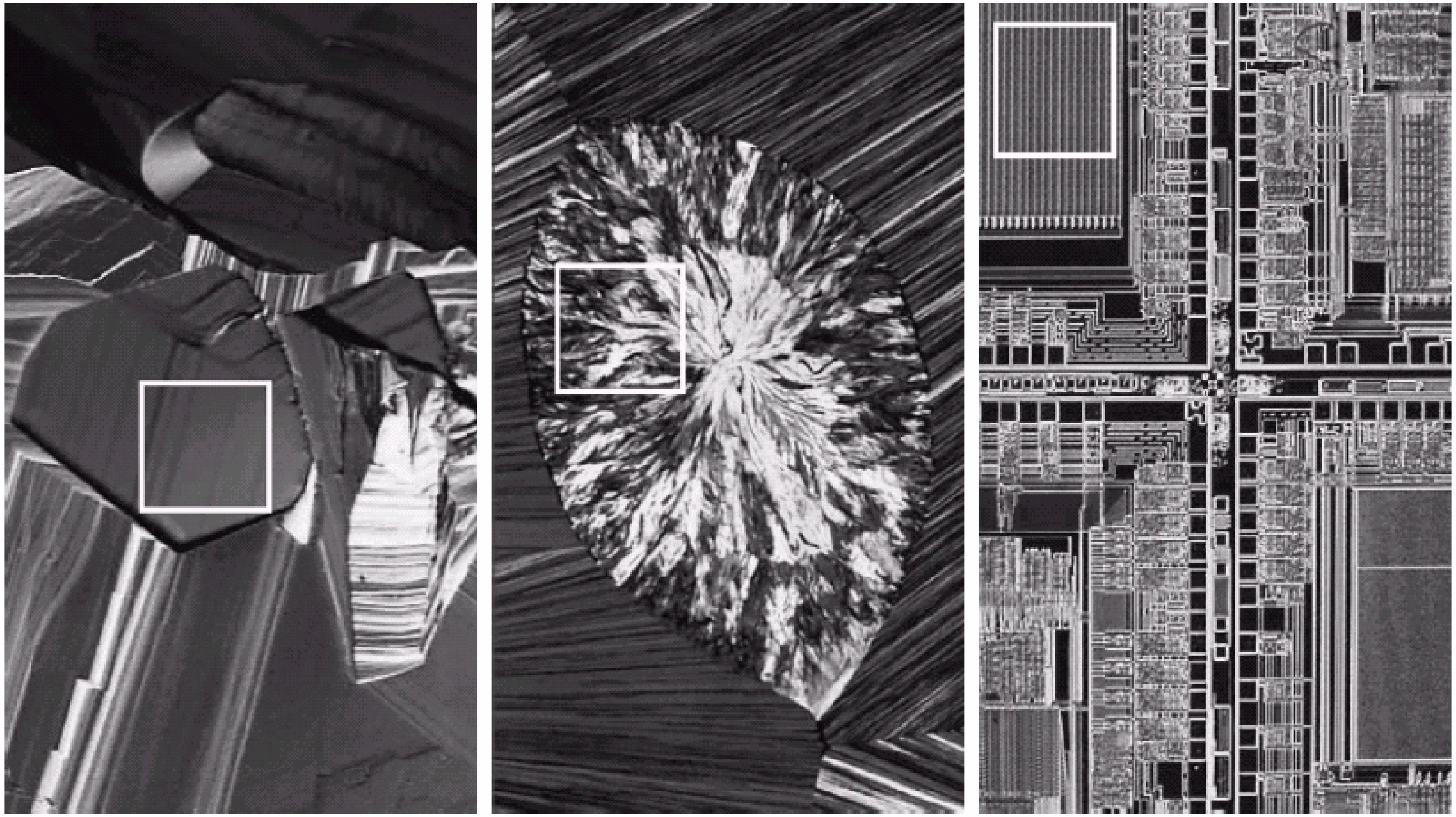
a b
c d

FIGURE 11.21 (a) Infrared image of the Washington, D.C. area. (b) Thresholded image. (c) The largest connected component of (b). Skeleton of (c).

Texture

- An important approach to region description is to quantify its *texture* content.

Chapter 11
Representation & Description



a b c

FIGURE 11.22 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

Statistical approaches

- The *n*th moment of *z* about the mean is

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n \, p(z_i) \qquad m = \sum_{i=0}^{L-1} z_i \, p(z_i)$$

- The second moment is of particular importance in texture description.
- It is a measure of gray-level contrast that can be used to establish descriptors of relative smoothness.
- For example, the measure is 0 for areas of contrast intensity (the variance is 0 here) and approaches 1 for large value of $\sigma^2(z)$.

$$R = 1 - \frac{1}{1 + \sigma^2(z)}$$

Statistical approaches

- The third moment $\mu_3(z) = \sum_{i=0}^{L-1} (z_i - m)^3 p(z_i)$ is a measure of the skewness of the histogram while the fourth moment is a measure of its relatives flatness.
- Some useful additional texture measures bases on histograms include a measure of "uniformity", given by $U = \sum_{i=0}^{L-1} p^2(z_i)$

  average entropy measure $e = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$

TABLE 11.2 Texture measures for the subimages shown in Fig. 11.22.

Texture	Mean	Standard deviation	*R* (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

Statistical approaches

- C is called the *gray-level co-occurrence matrix*
  - 1.Maximum probability $\max_{i,j}\left(c_{ij}\right)$
  - 2.Element difference moment of order k

$$\sum_i \sum_j (i-j)^k c_{ij}$$

  - 3.Inverse element difference moment of order k

$$\sum_i \sum_j c_{ij} \Big/ (i-j)^k \qquad i \neq j$$

  - 4.Uniformity $\sum_i \sum_j c_{ij}^2$
  - 5.Entropy $-\sum_i \sum_j c_{ij} \log_2 c_{ij}$

Chapter 11
Representation & Description

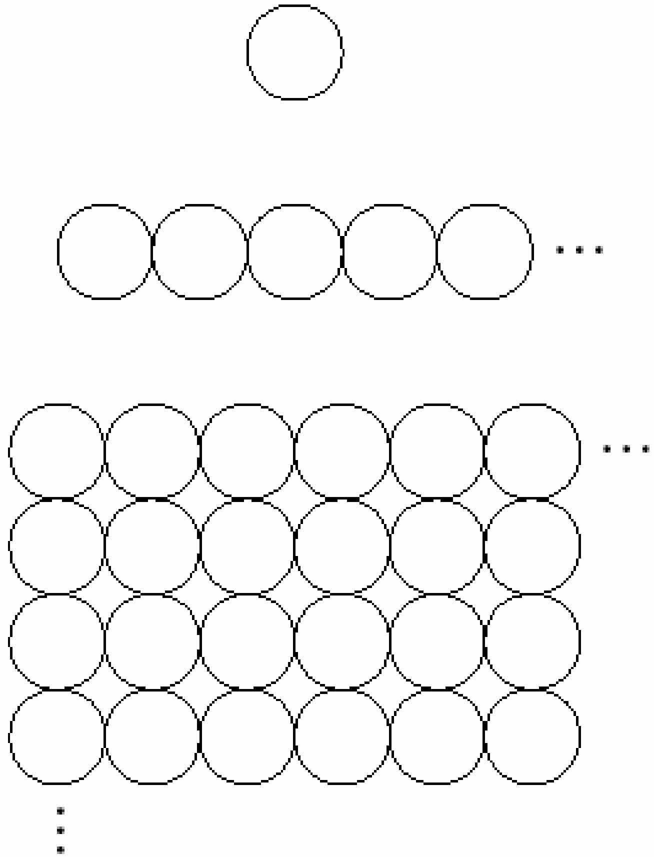
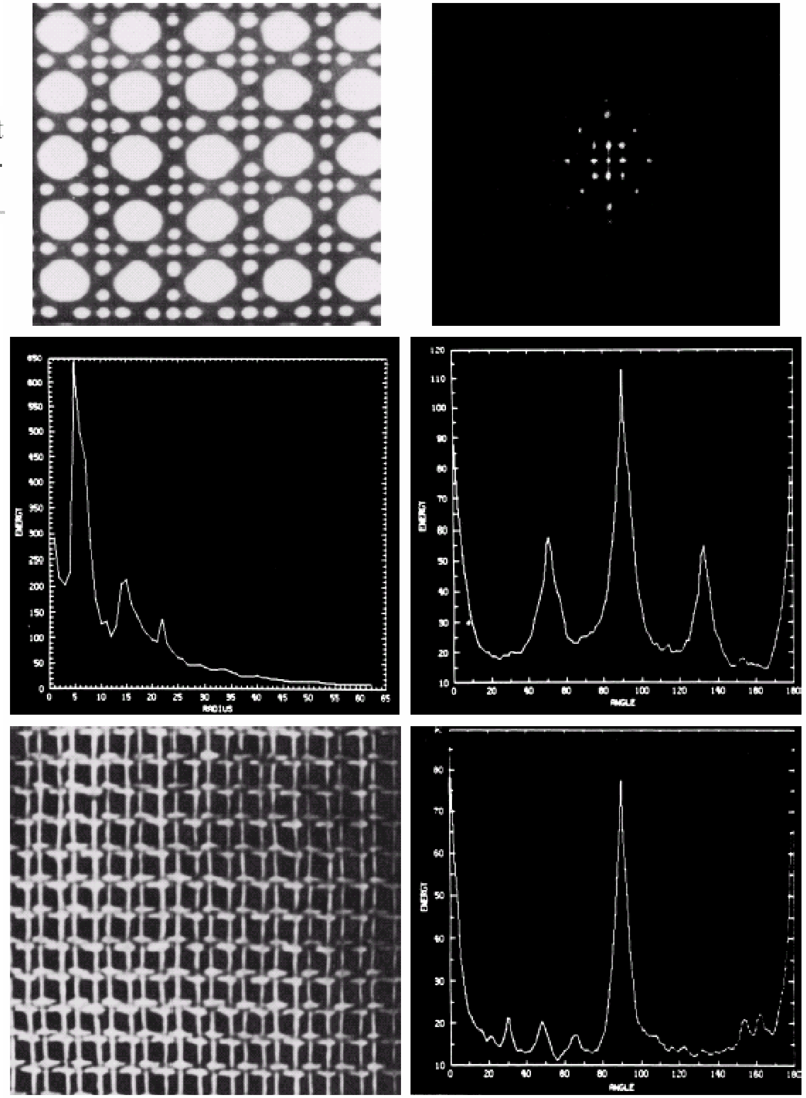


FIGURE 11.23 (a) Texture primitive. (b) Pattern generated by the rule $S \rightarrow aS$. (c) 2-D texture pattern generated by this and other rules.

Chapter 11
Representation & Description



a b
c d
e f

FIGURE 11.24 (a) Image showing periodic texture. (b) Spectrum. (c) Plot of $S(r)$. (d) Plot of $S(\theta)$. (e) Another image with a different type of periodic texture. (f) Plot of $S(\theta)$. (Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)

Use of Principal Components for Description

$$m_{\mathrm{x}} = \mathrm{E}\{\mathrm{x}\}$$

covariance matrix of the vector population is defined as

$$C_{\mathrm{x}} = \mathrm{E}\left\{(\mathrm{x} - \mathrm{m}_{\mathrm{x}})(\mathrm{x} - \mathrm{m}_{\mathrm{x}})^T\right\}$$

$$m_x = \frac{1}{K}\sum_{k=1}^{K} x_k$$

$$C_x = \frac{1}{K}\sum_{k=1}^{K} x_k x_k^T - \mathrm{m}_{\mathrm{x}}\mathrm{m}_{\mathrm{x}}^{\mathrm{T}}$$

Use of Principal Components for Description

$$y = A(x - m_x)$$

$$m_y = E\{y\} = 0$$

$$C_y = A C_x A^T$$

$$C_y = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

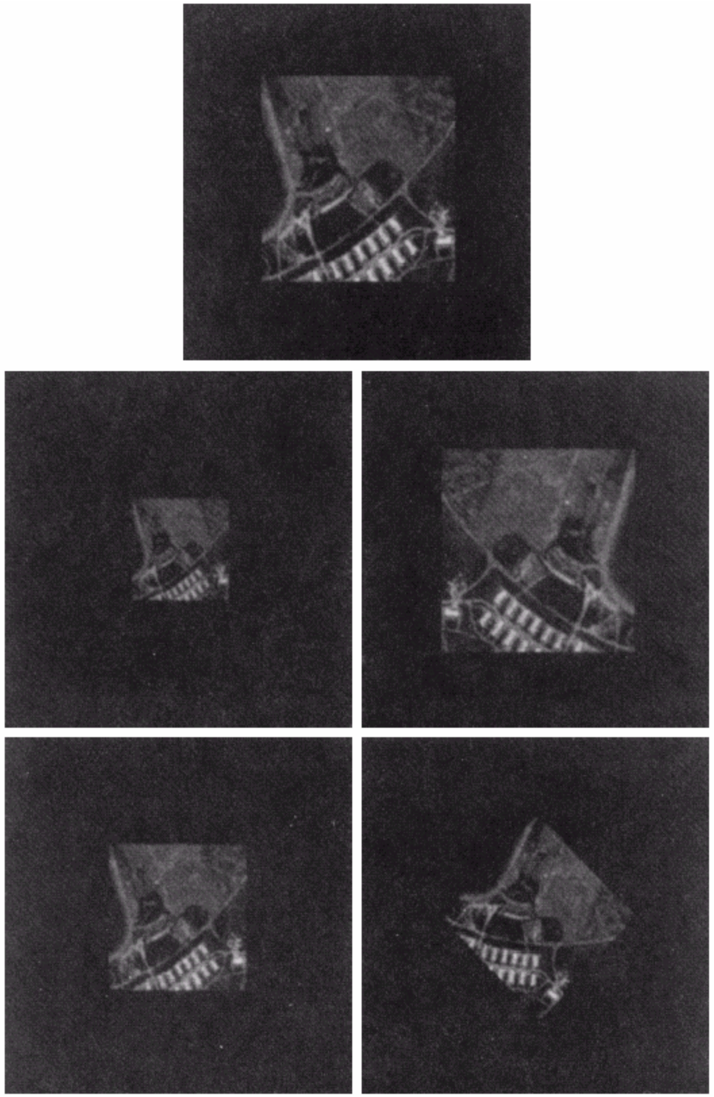
$$x = A^T y + m_x$$

$$\hat{x} = A_k^T y + m_x$$

$$e_{ms} = \sum_{j=1}^{n} \lambda_j - \sum_{j=1}^{k} \lambda_j$$

$$= \sum_{j=k+1}^{n} \lambda_j$$

Chapter 11
Representation & Description



a
b c
d e

FIGURE 11.25 Images used to demonstrate properties of moment invariants (see Table 11.3).

Chapter 11
Representation & Description

Invariant (Log)	Original	Half Size	Mirrored	Rotated 2°	Rotated 45°
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

TABLE 11.3 Moment invariants for the images in Figs. 11.25(a)-(e).

Chapter 11
Representation & Description

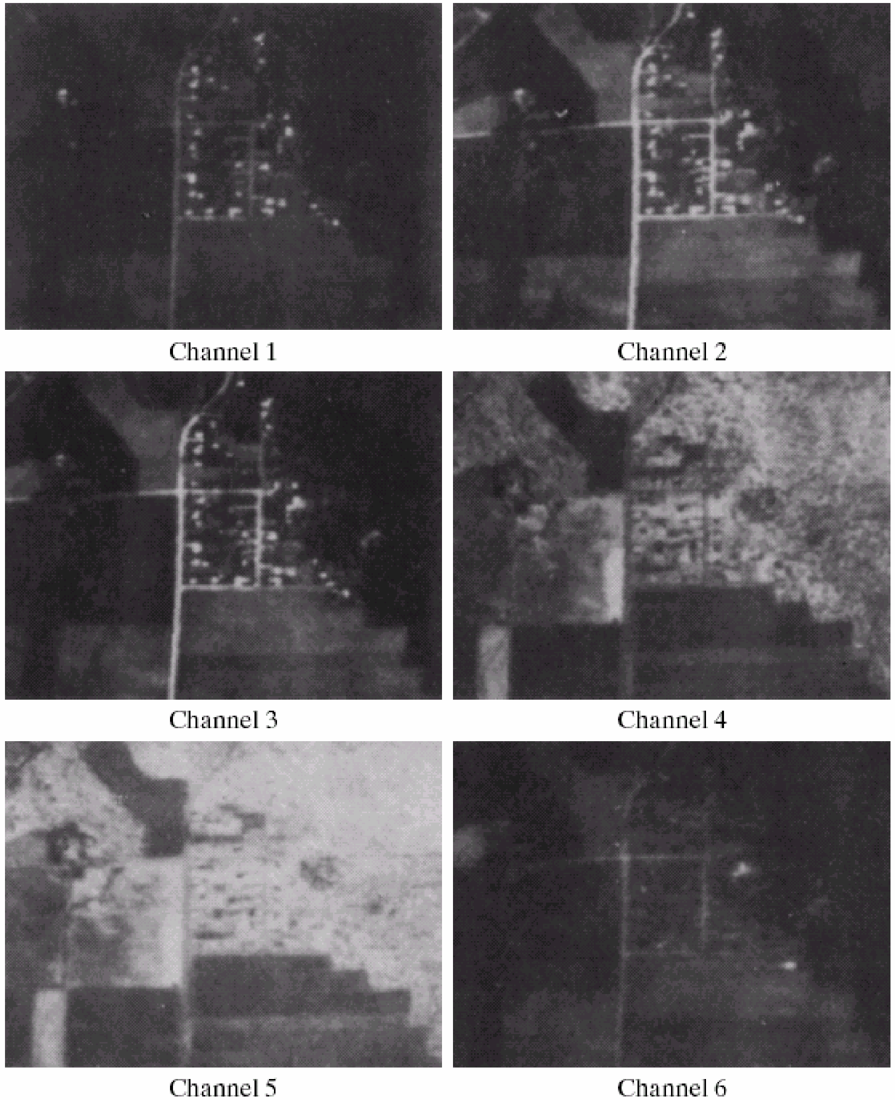


FIGURE 11.26 Six spectral images from an airborne scanner. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)

Chapter 11
Representation & Description

TABLE 11.4
Channel numbers and wavelengths.

Channel	Wavelength band (microns)
1	0.40–0.44
2	0.62–0.66
3	0.66–0.72
4	0.80–1.00
5	1.00–1.40
6	2.00–2.60

Chapter 11
Representation & Description

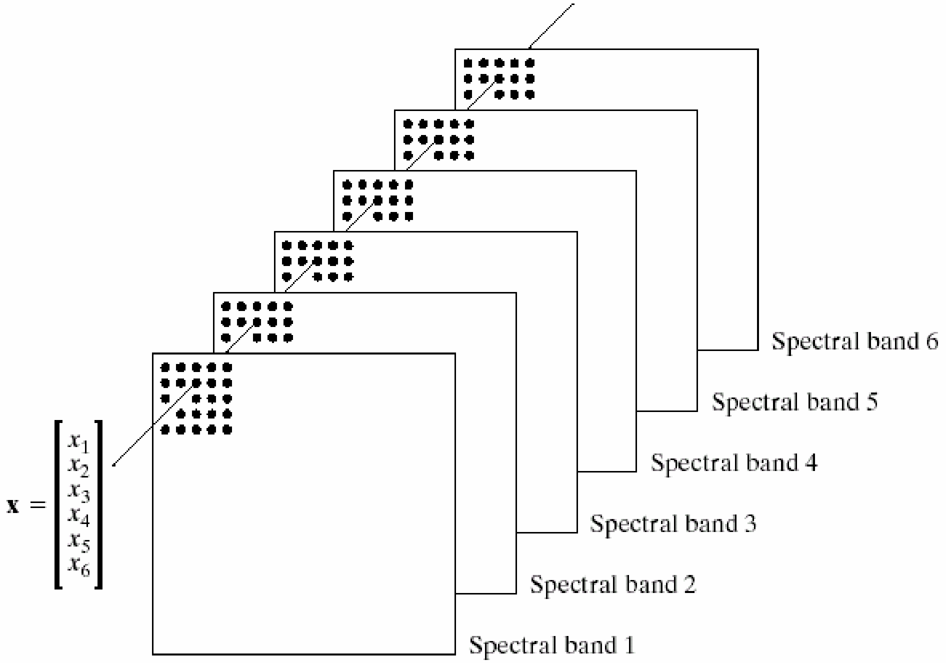


FIGURE 11.27 Formation of a vector from corresponding pixels in six images.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
3210	931.4	118.5	83.88	64.00	13.40

TABLE 11.5 Eigenvalues of the covariance matrix obtained from the images in Fig. 11.26.

Chapter 11
Representation & Description

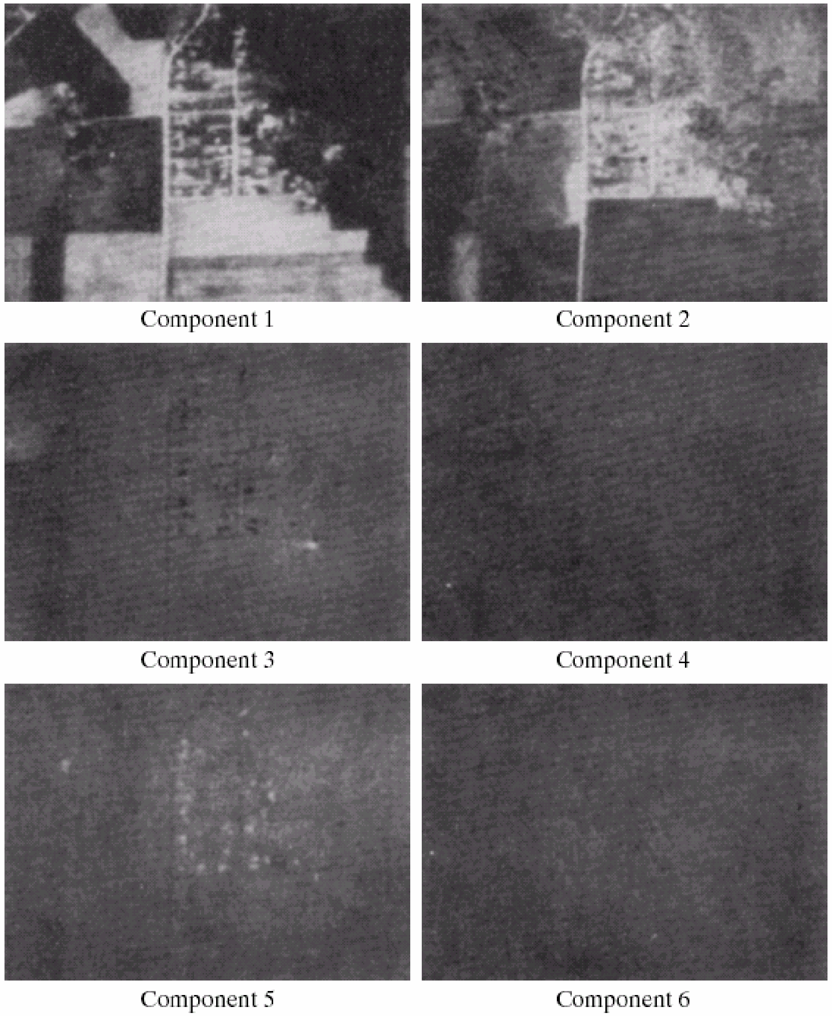


FIGURE 11.28 Six principal-component images computed from the data in Fig. 11.26. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)

Chapter 11
Representation & Description

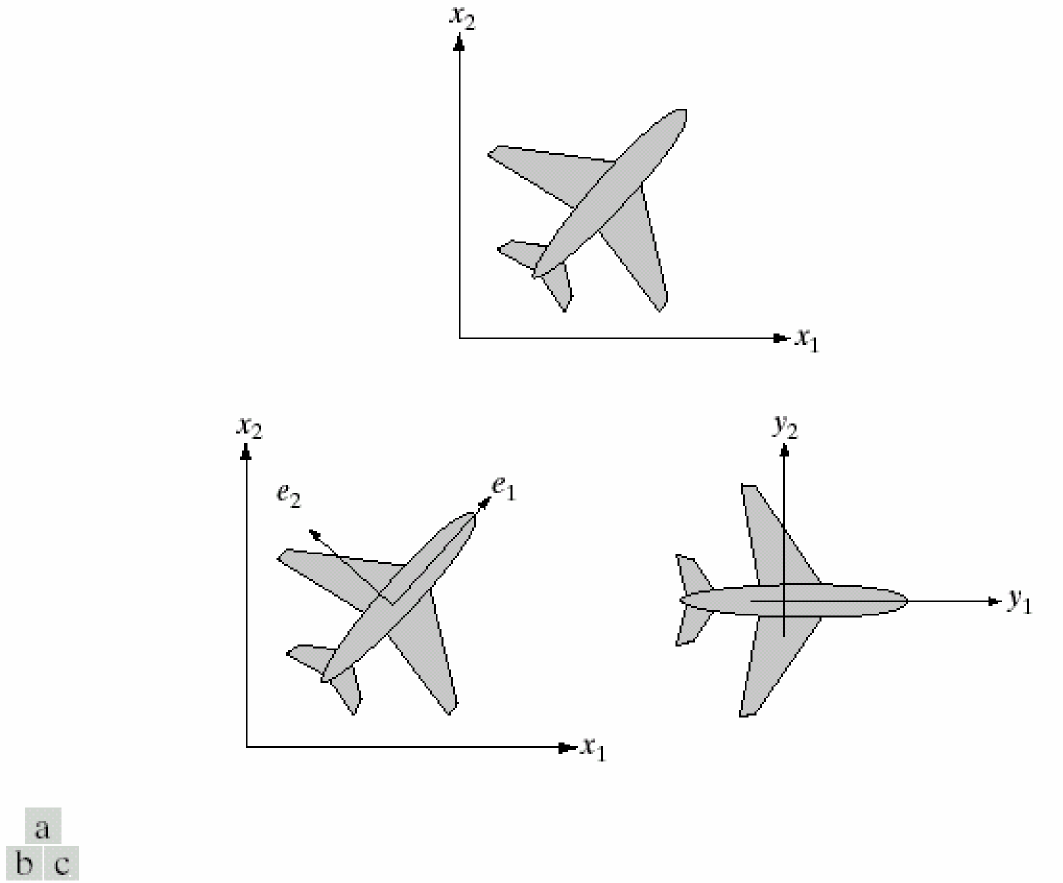
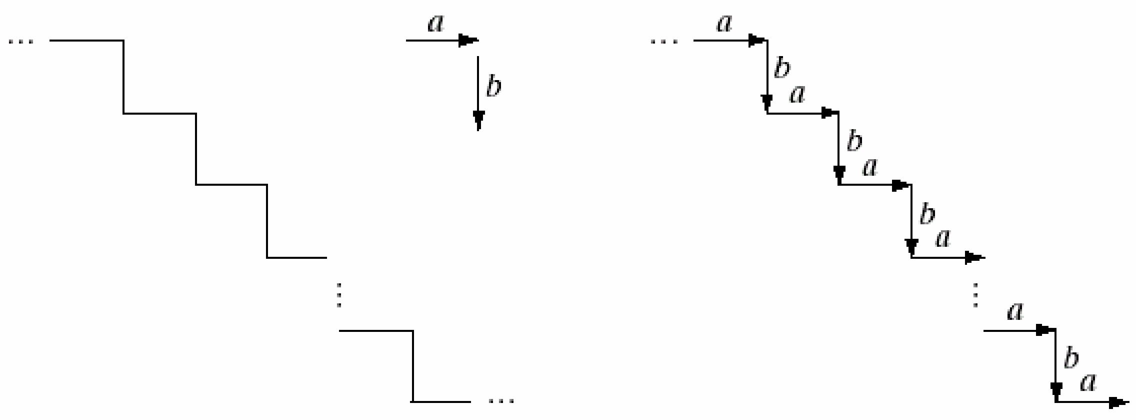


FIGURE 11.29 (a) An object. (b) Eigenvectors. (c) Object rotated by using Eq. (11.4-6). The net effect is to align the object along its eigen axes.

Chapter 11
Representation & Description

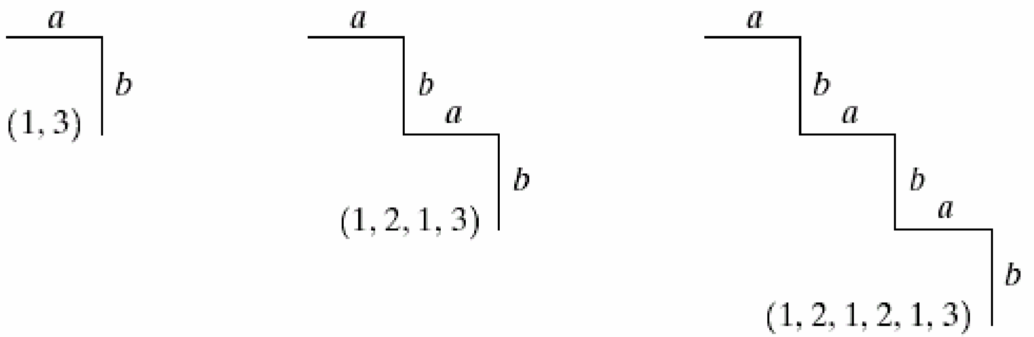


a b

FIGURE 11.30
(a) A simple staircase structure.
(b) Coded structure.

Chapter 11
Representation & Description

FIGURE 11.31 Sample derivations for the rules $S \rightarrow aA$, $A \rightarrow bS$, and $A \rightarrow b$.



Chapter 11
Representation & Description

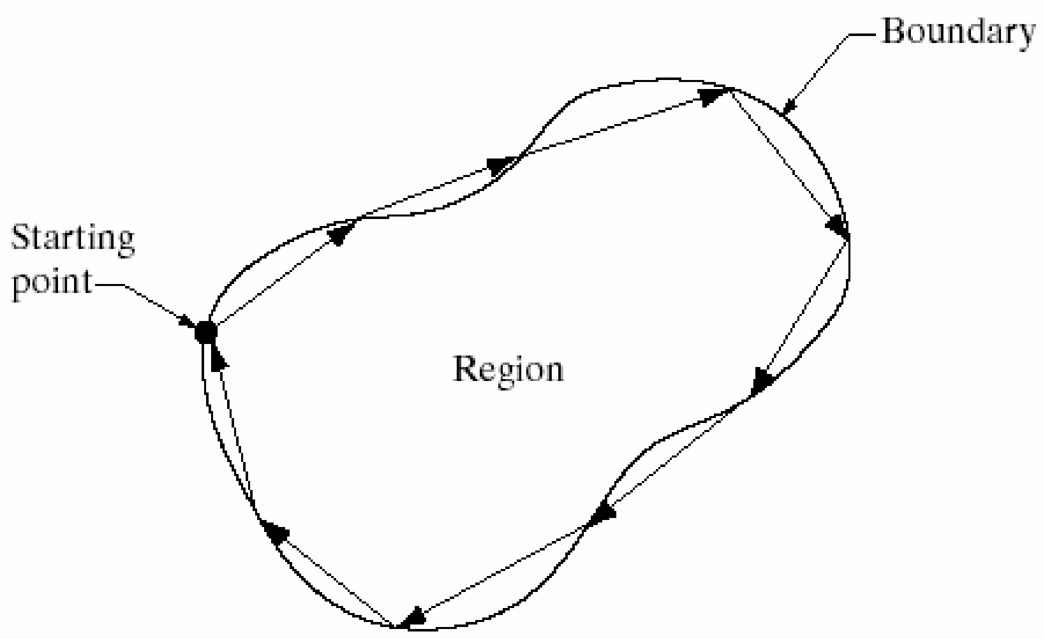
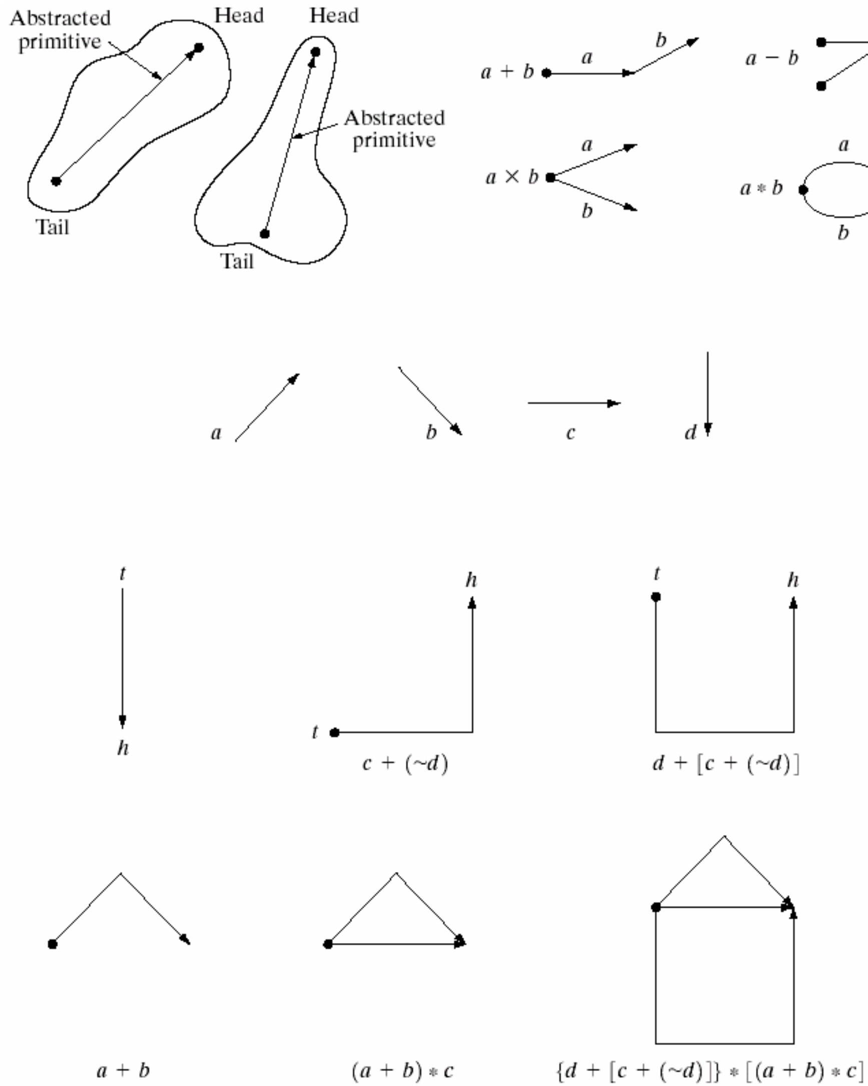


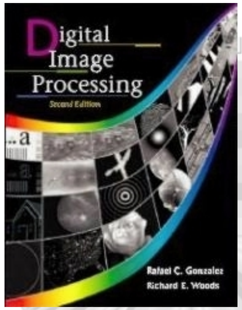
FIGURE 11.32 Coding a region boundary with directed line segments.

Chapter 11
Representation & Description



a b
c
d

FIGURE 11.33 (a) Abstracted primitives. (b) Operations among primitives. (c) A set of specific primitives. (d) Steps in building a structure.



Chapter 11
Representation & Description

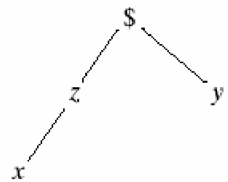


FIGURE 11.34 A simple tree with root \$ and frontier *xy*.

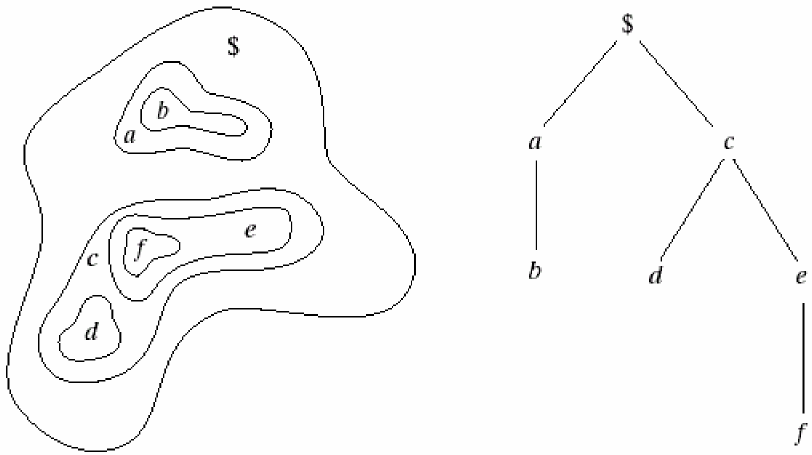


FIGURE 11.35 (a) A simple composite region. (b) Tree representation obtained by using the relationship "inside of."